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Power Spectra Robust Estimation of Ecg Signals Using Autoregressive Simulation

Abdul Khaleq A. Al-Naqeeb
College of Medical and Health Technology, Baghdad, Iraq
abdulkhaliq.alnaqeeb@yahoo.com

Ayad A. Ibrahim, Faez F. Shareef
University of Technology, Department of Electrical Eng., Baghdad, Iraq
Ayad.Asaad.Ibrahim@uotechnology.edu.iq; faezfawwaz@uotechnology.edu.iq

Abstract -Electrocardiogram (ECG) signal modeling is vital for the analysis and understanding of the biological consequences in patient's diagnosis. Both amplitude and spectral characteristics of these signals provide for normal and abnormal cases diagnosis. In this paper, a computer model, based on the autoregressive model parameters of a patient ECG signal is presented. For each ECG diagnosed case, the number of the parameters in the model is decided to use the Final Prediction Error procedure. The error signal (noise) which is the deviation of the simulated from the actual patient ECG signal is suitably filtered in order to obtain optimal diagnoses. Extensive series simulation studies show that the proposed technique provided far acceptable, dependable, robust model parameters calculation. Moreover, the signal power spectral shape estimates were obtained efficiently.

Keywords**:** robust estimation, power spectra, autoregressive (AR), simulated ECG, final prediction error (FPE).

1. Introduction

The ECG is a biomedical signal, arises whenever the human heart beats, and reflects its activity and performance of the human heart (Dliou et al., 2012). ECG features, such as shape and spectrum are of utmost importance as they are related to the patient cardiac health (Tamboli et al., 2013). Therefore, ECG modeling is necessary in relating the built model performance to that actually observed. ECG signals are non-stationary, many human physiological events and particular cardiac disorder can cause noticeable ECG signal variations (Saritha et al., 2008). In this work autoregressive based ECG signal is presented. The parameters are derived from a patient recorded ECG signals. The model will generate a simulated ECG signal whose amplitude and spectrum relate to the various patient physiological causes. Such simulation can be used to evaluate the signal performance of an ECG processing technique.

The initial estimates of AR, was obtained by applying the Sequential Least Square Method (SLSM), as one of the conventional methods. These estimates were compared to those obtained implementing this work suggested robust method. In order to obtain qualitative additions to the robust method in the field of fitting of the optimal distribution of residuals, a Probability Plot (PP) inspection formula is used, the plot is asymptotically to the normal distribution to estimate the standardized weighted value (q), that is done in order to obtain the fixed form of the new suggested distribution function (AL-Naqeeb, 1997).

2. Theory of the Optimal Filtering

Thomson, (1977a) pioneered the robust filtering method. In his work, the spectral density estimate of a signal is optimized according to a certain distribution pattern (Thomson, 1977b). AL-Naqeeb et al 1997 took the subject a step further by adding a weight function to the spectral density one. The weight function is generated according to a specific distribution asymptotic to the normal distribution function of the residuals.

ECG signal data in general contain outliers; therefore, obtained residuals in the AR model are no longer having normal distribution. Upper and lower outliers are reflected in the signal spectral density and sampling observation. These outliers are a result of filtering enhancement procedure called multi-contamination. Two types of contaminant distribution were introduced. There include the symmetric and asymmetric contaminant distributions.

2. 1. Symmetric Contaminant Distribution

If G (•) represents any symmetric distribution, which coincides around the center of the known symmetric distribution (F: N (μ, σ^2)), then G (•) will be (G: (μ, a σ^2)), while in case of shifting state. G (•) will be (G: ($\mu \pm b$, a σ^2)). According to the above basis, we could be chosen the symmetric alternative to the normal distribution such as Logistic, Double Exponential, and Cauchy in addition to Normal distribution.

2. 2. Asymmetric Contaminant Distribution

If G (•) represents any asymmetric distribution around any other point different from the location of the F (•) distribution, then the center point of G (•) will be different from (μ) and has a variance differs from that of F (•) (AL-Naqeeb, 2007). According to that, alternative asymmetric distributions have been chosen which extreme distributions, largest and smallest values were. The initial test of fitting the residual distributions ought to be done as an indicator to estimate the optimal weight function formula. It will be the standardized commutative density of residual distribution considered more fitting among the other standardized cumulative densities that are expected to the residual sets, which call the robust filtering. Table.1 show the suggested weighted filter formulas, which were generated according to the standardized cumulative density of specific distribution, derived by (AL-Naqeeb, 1997). Among different methods of fitting, the distribution of residual to estimates the grade of fitting or determining the level of significance for the critical points. For suitable test statistics and in order to give an accurate process of inspection, comparison and evaluation among different of suggested distributions. The probability plot method and relevant parameters were chosen, e.g. a single correlated coefficient measurement and goodness of fit according to the statistics as shown by (Smith. and Bain, 1976).

Table. 1. The new suggested weighting of fitting distribution functions

Adjusted Distribution Functions	***Filter Weighted Function***
Adjusted Extreme value-Smallest	$w(u) = \exp\left[-\exp(-(q(\lvert u\rvert - q)))\right]$
Adjusted Extreme value-Largest	$w(u) = \exp\left[-\exp((q(\lvert u\rvert - q)))\right]$
Adjusted Cauchy	$w(u) = 0.5 + \pi^{-1}\tan^{-1}(q(\lvert u\rvert - q))$
Adjusted Logistic	$w(u) = 1/\left[1+\exp\left[-(q(\lvert u\rvert - q))\right]\right]$
Adjusted Double Exponential	$w(u) = \begin{cases} 1 - 0.5\exp\left[-(q(\lvert u\rvert - q))\right. \\ 0.5\exp\left[q(\lvert u\rvert - q)\right] : o.w. \end{cases}$ *if* $u \geq a$
Adjusted Normal	$w(u) = 0.5\left[1+\left\{1-\exp(-2(q(\lvert u\rvert - q))^2/\pi)\right\}^{0.5}\right]$

Where:

u: is the residual random variable.

q: is a constant = $\phi^{-1}{}_{N(0,1)}(1/N)$.

$\phi_{N(0,1)}$: normal cumulative density function.
N: No. of samples size per interval.

$$r_{(R_{(t)}.K_t)} = \frac{\sum_{t=p-1}^{n}(K_t - K)(R_{(t)} - R)}{\left(\sum_{t=p-1}^{n}(K_t - K)^2 \sum_{t=p-1}^{n}(R_{(t)} - R)^2\right)^{0.5}} \quad (1)$$

$$S_R = 1 - r^2(R_{(t)}.K_t) \quad (2)$$

Where:

R $_{(t)}$: order statistics of standardized residual according to the scale parameter of the assumed distribution.

K: inverse probabilities of the cumulate sample distribution related to assumed distribution.

2. 3. The Studied Distributions

The suggested formula of probability plot method is presented in Table. 2. It represents the inverse probabilities of a cumulative sampling of normal and non-normal distribution functions. The proposed formula is more accurate than others formula that shown by (AL-Naqeeb, 2010), as follow:

$$F^{-1}((t-0.5)/n) \quad (3)$$

The order statistics of the standardized residuals are illustrated by:

$$R_{(r)} = \left(\frac{\varepsilon_{(t)}}{Sp}\right) \quad ; \text{Where Sp = Scale Parameter. Ref. (AL-Naqeeb, 1997, 2007).} \quad (4)$$

Table. 2. The suggested inverse probabilities of the cumulative sampling distribution functions

Distribution Functions	$F^{-1}((t-0.5)/n)$
Extreme value-Smallest	$Ln(Ln(n/(n-t+0.5)))$
Extreme value-Largest	$-Ln(Ln(n/(n-t+0.5)))$
Cauchy	$\tan(\pi((2t-1-n)/2$
Logistic	$Ln((t-0.5)/(n-t+0.5))$
Double Exponential	$Ln(2t-1)/n)$
Normal (*)	$((\pi/2)(-Ln(1-((2t-n-1)/n)\text{^}2)))\text{^}0.5$

(*) Polya approximation formulae are applied to solve integration of Normal distribution function, and derived by (Polya, 1945).

Since the standardized cumulative density-function cannot evaluate directly to obtain $F^{-1}(u)$ as a function of (w); where (w) is a random variable.

$$F(u) = (2\pi)^{-0.5} \int_{-a}^{u} \exp\left(-\frac{1}{2} w^2\right) dw \tag{5}$$

One of the approximate formulae to solve the equation above was chosen:

$$F(u) \approx 0.5\left(1 + \left(1 - \exp\left(-2u^2 / 2\pi\right)\right)^{0.5}\right) \tag{6}$$

Where at u=1.6 maximum error ≈ 0.003. From the above basis, a new formula was used for the inverse probability cumulative sample of the normal distribution derived by (Polya, 1945) as shown below:

Let F (u) =Y

$$u = \left(\pi / 2\left(-\ln\left(1 - (2y-1)^2\right)\right)\right)^{0.5} \tag{7}$$

$$F^{-1}(u) = \left((\pi / 2)\left(-\ln\left(1 - (2u-1)^2\right)\right)\right)^{0.5} \tag{8}$$

$$F^{-1}\left(\frac{t}{n+1}\right) = \left((\pi / 2)\left(-\ln\left(1 - \left(\frac{2t-n-1}{n+1}\right)^2\right)\right)\right)^{0.5} \tag{9}$$

Finally, The Inverse Probabilities of Normal cumulate sampling distributions Functions are given by:

$$F^{-1}\left(\frac{t-0.5}{n}\right) = \left((\pi / 2)\left(-\ln\left(1 - \left(\frac{2t-n-1}{n}\right)^2\right)\right)\right)^{0.5} \tag{10}$$

In summarizing of the preceding process, fig. 1 illustrated the flowchart of the proposed technique algorithm.

3. Experimental Set-up for ECG Measurements

The experimental setup is shown in the block diagram of fig. 1**.** A normal, and (Segment Depression-KH, Wandering Atrial Pacemaker and Acute Anterior) abnormal of four subjects for each were investigated. The recording system consisted of a pair of gold surface electrodes, which are built on a single module with a high differential gain amplifier [AD620]. It has a voltage gain of about 950, and a high common mode rejection ratio (typically 90 dB at 50 Hz).

For each subject, the ECG signal was sampled at 400 Hz by using ARDUINO microcontroller as an **10**–bit analogue to digital convertor, and connect to a laptop via USB cable. The sampling rate used is quite sufficient (Castiglioni, et al., 2003). This was performed for each case.

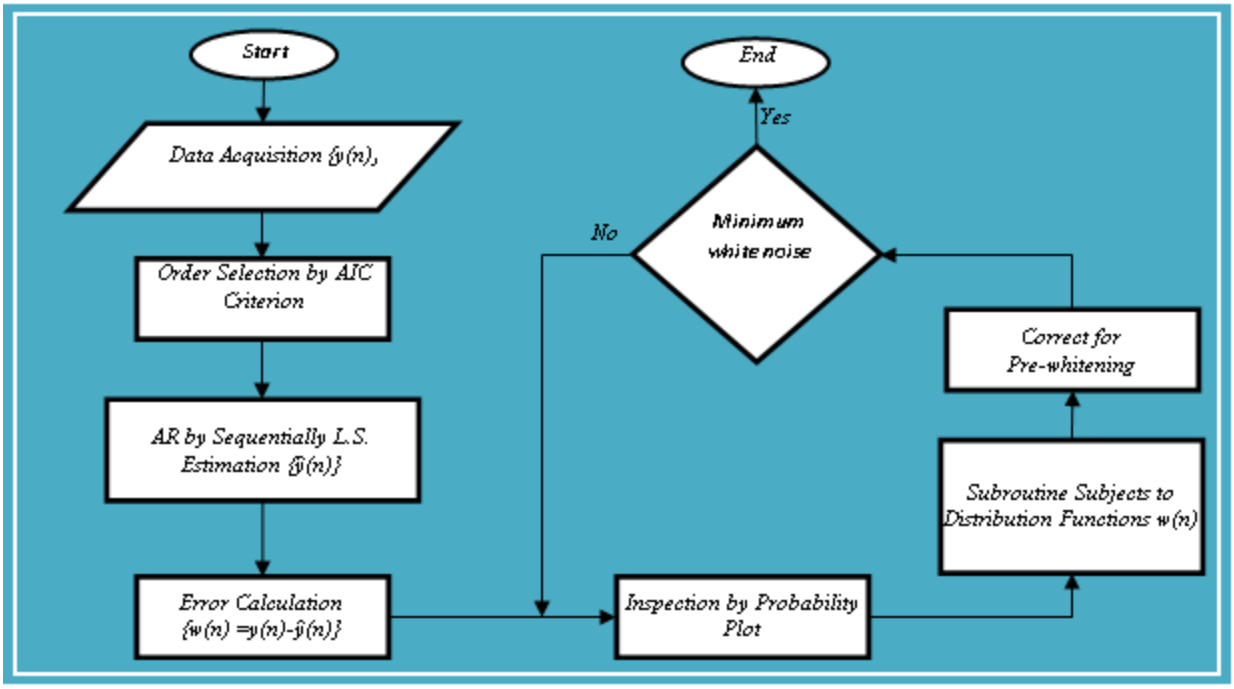


Fig. 1. Flowchart of the proposed technique

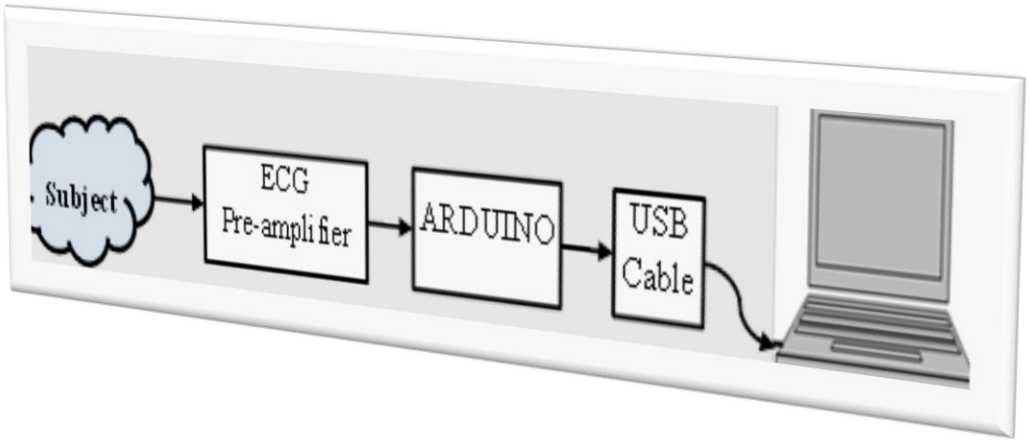


Fig. 2. ECG Experimental set-up

4. Results

The model was tested on four subjects for each case. Our intention is to combine two processors in a single processor that hopefully gives better results. In The first, is to find order of the (AR) model (P). In the second, we pass the error signal w (u), which is the difference between the real and simulated signals, through the suggested distribution filters.

4. 1. The Autoregressive Model (P)

The order (P) of the autoregressive model AR, mean that the present sample is given as a weighted sum of previous samples, lagged by the model order (P), this were estimated by using the final predicted error (FPE) suggested by (Akaike, 1969). Table. 3 shows the order of the model for each subject and case.

Table. 3. The order of the model (P)

ECG diagnosed case	*Subject 1*	*Subject 2*	*Subject 3*	*Subject 4*
Normal	6	7	6	5
Segment Depression-KH	20	20	20	20
Wandering Atrial Pacemaker	10	9	10	10
Acute Anterior	15	17	17	17

As shown in the table. 3, the model order (P) for each case can be in general discriminating between the normal and the other abnormal cases, the high variance between different orders is related to the nature of the real signals. Thus, when the ECG corresponding to the case applies to this process, the response is the order of the model order (P), which indicates the diagnosed case.

4. 2. Pre-whitening the Error Signal *w(u)*

The uses of the robust optimality filtering to pre-whitening the errors signal *w(u)*, by choosing the proper distribution model . These models were deemed necessary because of the different contamination fields, which reflect different shapes of initial values of the error signals *w(u).*

These initial values might be distributed in the light of upstairs or downstairs outliers occurred or within the sampling observations. Therefore, it is necessary to distinguish between two kinds of contaminated distributions, symmetrical and asymmetrical distributions. Two symmetrical distributions were chosen instead of the normal distribution, which are extreme value distribution, i.e. smallest (E-) and largest (E+) values. While for the symmetrical distribution, chose Cauchy (C), Logistic (L), double exponential (D) distribution as the normal (N) distribution. Table. 4 shows the responses of each filter (adjusted distribution) by considering the number of iterations to pre-whitening the errors of signal *w(u).* In all cases and subjects, this leads us to an important conclusion, that the significance of the weighted filtering functions is related to the symmetrical distribution, known by Cauchy distribution.

Table. 4. The robust optimality filtering of the *w(u)*

ECG diagnosed case	*Distribution*					
	E+	*E-*	*C*	*L*	*D*	*N*
Normal	1	9	10	7	0	3
Segment Depression-KH	1	9	10	7	0	3
Wandering Atrial Pacemaker	1	9	10	7	0	3
Acute Anterior	1	9	10	7	0	3

Fig. 3 shows the power spectrum of the initial w (u) and the final iteration process of the suggested smoothing technique, related to Cauchy the symmetrical distribution, for diagnosing a case (Wandering Atrial Pacemaker), which was selected arbitrarily.

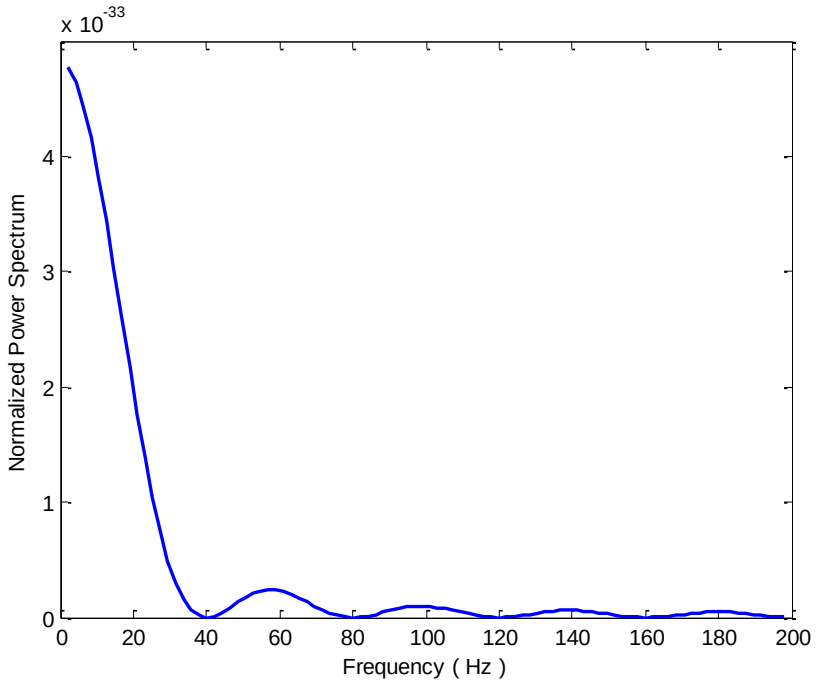
5. Conclusions

In this work, an accurate technique for diagnosing the ECG patient signals is presented. The proposed technique shows a high sensitivity and dependence of the diagnoses of the normal and abnormal patient cases.

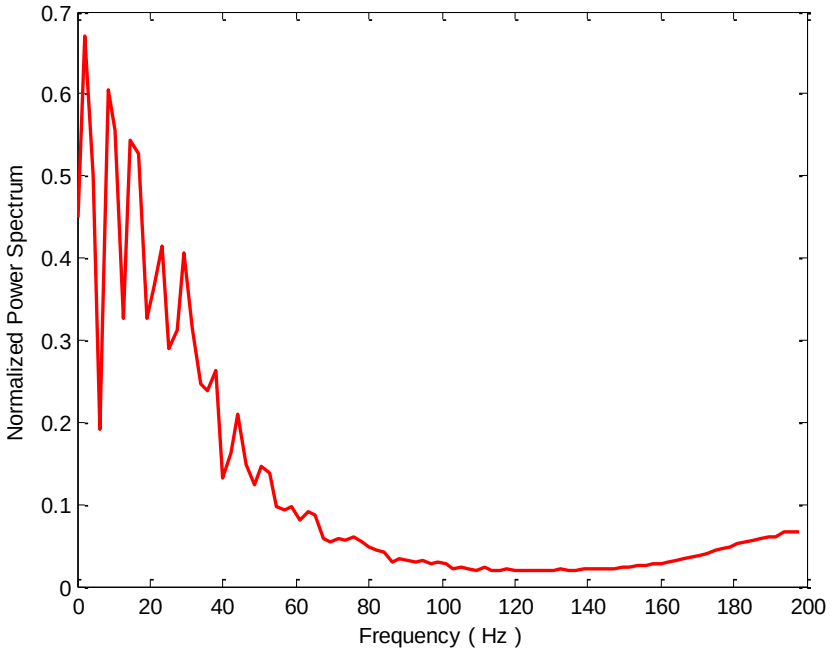
The variance of the autoregressive model order presented in table. 3 shows clearly the high selectivity using the proposed technique. That of course reflects on the patient diagnosed case.

All normal and abnormal patients ECG error signal *w(u)* after filtering were all seen related to the adjusted Cauchy distribution function.

The model in its final form can generate any desired length of ECG signal for different types of diagnosed cases, and can be used as test signals for further processing.



(a)



(b)

Fig. 3. The power spectrum of (a- error signal *w(u)* and b- final iteration) using Cauchy distribution

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